

EXERCISE 2
MODULAR FORMS 2019
DUE DATE: MARCH 31, 2019

Exercise 1. Let $L \subset \mathbb{C}$ be a lattice, and $f(z)$ be an elliptic function for L , that is a meromorphic function so that $f(z + \omega) = f(z)$ for all $\omega \in L$. Assume that f is analytic except for double poles at each point of the lattice L . Show that $f = a\wp + b$ for some constants $a \neq 0$, b .

We defined the Eisenstein series $G_k(L) = \sum_{0 \neq \omega \in L} 1/\omega^k$, $k > 2$.

Exercise 2. Show that the Weierstrass $\wp$ function satisfies

$$\wp''(z) = 6\wp(z)^2 - \frac{1}{2}g_2$$

where $g_2 = 60G_4$.

Exercise 3. Show that

$$G_8 = \frac{3}{7}G_4^2$$